# Correlating Viscosity in Urea-Formaldehyde Polymerization 

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#### Abstract

The objective of this study was to find a correlation for the viscosity changes during step polymerization of urea and formaldehyde. For this reason, a kinetic model for the condensation reactions was proposed, and using this model, a correlation for the kinematic viscosity changes of the solution during polymerization was obtained. Viscosity measurements of the samples during condensation reactions were carried out using Ubbelohde viscometers at intervals. The experimental data were curve fitted using our model, which was found to correspond very well. © 1997 John Wiley \& Sons, Inc. J Appl Polym Sci 69: 631-636, 1997


Key words: urea; formaldehyde; viscosity; polymerization; kinetic

## INTRODUCTION

Urea-formaldehyde is mainly used as an adhesive in plywood and particle board industries. Two main steps are recognized in the reaction of formaldehyde with urea. The first is a simple addition of formaldehyde to introduce the hydroxymethl group and is catalyzed by either acids or bases. Hence, it takes place over the entire pH range. ${ }^{1-12}$ The second step is a condensation reaction that involves the linking together of monomer units with the liberation of water to form a dimer, a polymer, or a vast network. The second reaction is only catalyzed by acids.

Tomita and Hirose ${ }^{13}$ have shown that for form-aldehyde-to-urea ( $\mathrm{F} / \mathrm{U}$ ) molar ratios between 1.7 and 2.2 within the pH range $4-9$, the following reactions take place:


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where UF, FUF, UFF, and FUFF are monomethylolurea, $N, N^{\prime}$-dimethylourea, $N, N$-dimethylolurea, and trimethylolurea, respectively.

This system was successfully observed by $H$ NMR nuclear magnetic resonance spectroscopy on decomposition of both monomethylolurea and $N, N^{\prime}$-dimethylolurea. ${ }^{13}$ Methylolureas so formed undergo a series of condensation reactions with urea and themselves, giving rise to a series of products. These condensation reactions have been the subject of extensive studies, and several articles have been reported in the literature. ${ }^{14-18}$ The following reaction scheme has been proposed, where $M$ is the methylene bridge:

$$
\begin{align*}
\mathrm{UF}+\mathrm{U} & \rightarrow \mathrm{UMU}  \tag{1}\\
\mathrm{UF}+\mathrm{UF} & \rightarrow \mathrm{UMUF}  \tag{2}\\
\mathrm{UF}_{2}+\mathrm{UF}_{2} & \rightarrow \mathrm{FUMUF}_{2}  \tag{3}\\
\mathrm{UF}_{2}+\mathrm{U} & \rightarrow \mathrm{UMUF}  \tag{4}\\
\mathrm{UF}_{2}+\mathrm{UF} & \rightarrow \mathrm{FUMUF} \tag{5}
\end{align*}
$$

It has been shown that the reaction between the amido methylol group and an amino group is bimolecular and hydrogen ion catalyzed. How-
ever, no buffer influence has been observed. The rate constant has been found to depend on the type of amide group or the amido methylene group. ${ }^{18}$

In this study, we have used the functional group approach in which we follow the polymerization by defining five reactive species. On the basis of this approach, a mathematical model is developed to predict the variation of polymer viscosity versus time, as discussed below.

## Kinetic Model for the Formation of Urea-Formaldehyde Polymer

According to the work of Smets and Borzee ${ }^{19}$ on the condensation of monomethylolureas, both thermal and acid-catalyzed reactions occur together, and the rate equation is as follows.

$$
\begin{equation*}
-d[x] / d t=K\left[H^{+}\right][M]^{2}+K^{\prime}[M]^{2} \tag{6}
\end{equation*}
$$

where $[M],[X]$, and $\left[H^{+}\right]$are concentrations of monomethylolurea, methylol group, and hydronium ion, respectively. $K$ and $K^{\prime}$ are rate constants.

In view of this, the following reaction mechanism has been proposed; eq. (9) is the rate-limiting step.


Since the backward and forward rate constants of eqs. (7) and (8) are high with respect to eq.
(9), we may assume that they are at equilibrium. Let us apply the following notation:


Thus, we can write the above reaction mechanism as follows in which $i$ and $j$ denote the number of mono-, di-, or trimethylol molecules attached together.

$$
\begin{align*}
& M_{i}+\mathrm{H}^{+} \underset{\stackrel{k_{1}^{\prime}}{\rightleftarrows}}{\underset{k_{1}}{\rightleftharpoons}} M_{i} \mathrm{H}^{+}  \tag{14}\\
& M_{i} \mathrm{H}^{+} \stackrel{k_{2}}{\stackrel{k_{2}^{\prime}}{\rightleftharpoons}} M_{i}^{\prime+}+\mathrm{H}_{2} \mathrm{O}  \tag{15}\\
& M_{i}^{\prime+}+M_{j} \xrightarrow{k_{3}} M_{i+j}^{+}  \tag{16}\\
& M_{i+j}^{+} \stackrel{k_{4}}{\rightarrow} M_{i+j}+\mathrm{H}^{+} \tag{17}
\end{align*}
$$

$M_{i}^{\prime{ }^{+}}$species can react with all $M_{j}$, but $M_{i}^{+}$cannot.
The reaction rate of different groups involved in the reaction are not the same, thus, $k_{3}$ not only is a function of temperature but also $\mathrm{F} / \mathrm{U}$ ratio. In other words, $k_{3}$ is an average, the value of which is assumed constant within the measured viscosity range. Moreover, there is a thermal reaction path which is bimolecular, according to the work of Smets and Borzee. ${ }^{19}$

$$
\begin{equation*}
M_{i}+M_{j} \xrightarrow{k_{t}} M_{i+j}+\mathrm{H}_{2} \mathrm{O} \tag{18}
\end{equation*}
$$

Here, $k_{t}$ is also a rate constant that is a function of temperature and the $\mathrm{F} / \mathrm{U}$ ratio.

Considering the proposed mechanism, we can write the rate equation for different species as follows:

$$
\begin{align*}
\frac{d\left[M_{1}\right]}{d t}= & -k_{3}\left[M_{1}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right] \\
& \quad-k_{t}\left[M_{1}\right] \sum_{i=1}^{\infty}\left[M_{i}\right]  \tag{19}\\
\frac{d\left[M_{2}\right]}{d t}= & -k_{3}\left[M_{2}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right]+k_{4}\left[M_{2}^{+}\right] \\
& +\frac{k_{t}}{2}\left[M_{1}\right]^{2}-k_{t}\left[M_{2}\right] \sum_{i=1}^{\infty}\left[M_{i}\right]  \tag{20}\\
\frac{d\left[M_{3}\right]}{d t}= & -k_{3}\left[M_{3}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right]+k_{4}\left[M_{3}^{+}\right] \\
& +\frac{k_{t}}{2}\left[M_{1}\right]\left[M_{2}\right]+\frac{k_{t}}{2}\left[M_{2}\right]\left[M_{1}\right] \\
\frac{d\left[M_{4}\right]}{d t}= & -k_{3}\left[M_{4}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right]+k_{4}\left[M_{4}^{+}\right]  \tag{21}\\
& +\frac{k_{t}}{2}\left[M_{1}\right]\left[M_{3}\right]+\frac{k_{t}}{2}\left[M_{2}\right]\left[M_{2}\right] \\
& +\frac{k_{t}}{2}\left[M_{3}\right]\left[M_{1}\right]-k_{t}\left[M_{4}\right] \sum_{i=1}^{\infty}\left[M_{i}\right]
\end{align*}
$$

The factor of $\frac{1}{2}$ appears in these expressions because the rate constant $k_{t}$ is defined with respect to the rate of disappearance of the polymer molecules (two molecules of $M_{1}$ disappear for each molecule of $M_{2}$ formed in the reaction $M_{1}+M_{1} \rightarrow M_{2}$ ).

Since the reactions $M_{1}+M_{2} \rightarrow M_{3}$ and $M_{2}+M_{1} \rightarrow M_{3}$ are indistinguishable from each other, it follows that the rate of appearance of polymer $M_{3}$ by these thermal reactions may be represented as

$$
\begin{equation*}
\frac{d\left[M_{3}\right]}{d t}=\frac{k_{t}}{2}\left[M_{1}\right]\left[M_{2}\right]+\frac{k_{t}}{2}\left[M_{2}\right]\left[M_{1}\right] \tag{23}
\end{equation*}
$$

Thus, the overall rate of appearance of any poly$\operatorname{mer} M_{P}$ is

$$
\begin{align*}
\frac{d\left[M_{p}\right]}{d t}= & -k_{3}\left[M_{p}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right] \\
& +k_{4}\left[M_{p}^{+}\right]+\frac{k_{t}}{2} \sum_{i=1}^{P-1}\left[M_{i}\right]\left[M_{p-i}\right] \\
& \quad-k_{t}\left[M_{p}\right] \sum_{i=1}^{\infty}\left[M_{i}\right] \tag{24}
\end{align*}
$$

Summing up the above equations yields

$$
\begin{align*}
& \frac{d \sum_{j=1}^{\infty}\left[M_{j}\right]}{d t}=-k_{3} \sum_{j=1}^{\infty}\left[M_{j}\right] \sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right] \\
& +k_{4} \sum_{j=2}^{\infty}\left[M_{j}^{+}\right]+\frac{k_{t}}{2} \sum_{j=2}^{\infty} \sum_{i=1}^{j-1}\left[M_{i}\right]\left[M_{j-i}\right] \\
& \\
& \quad-k_{t} \sum_{j=1}^{\infty}\left[M_{j}\right] \sum_{i=1}^{\infty}\left[M_{i}\right] \tag{25}
\end{align*}
$$

Since eqs. (14) and (15) are fast and at equilibrium, it follows that

$$
\begin{align*}
{\left[M_{i}^{\prime+}\right] } & =\frac{k_{2}\left[M_{i} H^{+}\right]}{k_{2}^{\prime}\left[\mathrm{H}_{2} \mathrm{O}\right]}  \tag{26}\\
{\left[M_{i} \mathrm{H}^{+}\right] } & =\frac{k_{1}}{k_{1}^{\prime}}\left[M_{i}\right]\left[\mathrm{H}^{+}\right] \tag{27}
\end{align*}
$$

Because of a large amount of water in the reaction media, we may assume that its concentration is constant, so we have

$$
\begin{align*}
{\left[M_{i}^{\prime+}\right] } & =\frac{k_{2} \cdot k_{1}\left[M_{i}\right]\left[\mathrm{H}^{+}\right]}{k_{2}^{\prime} \cdot k_{1}^{\prime}\left[\mathrm{H}_{2} \mathrm{O}\right]}=K_{e}\left[M_{i}\right]\left[\mathrm{H}^{+}\right]  \tag{28}\\
K_{e} & =\frac{k_{2} \cdot k_{1}}{k_{2}^{\prime} \cdot k_{1}^{\prime}\left[\mathrm{H}_{2} \mathrm{O}\right]} \tag{29}
\end{align*}
$$

When the expression given by eq. (28) is summed over all $i(i=1,2,3, \cdots)$, the following result is obtained:

$$
\begin{equation*}
\sum_{i=1}^{\infty}\left[M_{i}^{\prime+}\right]=K_{e}\left[\mathrm{H}^{+}\right] \sum_{i=1}^{\infty}\left[M_{i}\right] \tag{30}
\end{equation*}
$$

Considering eq. (28), we can write the rate equations for intermediate species $M_{j}^{+}$'s:

$$
\begin{align*}
& \quad \frac{d\left[M_{2}^{+}\right]}{d t}=+\frac{k_{3}}{2} K_{e}\left[\mathrm{H}^{+}\right]\left[M_{1}\right]^{2}-k_{4}\left[M_{2}^{+}\right]  \tag{31}\\
& \frac{d\left[M_{3}^{+}\right]}{d t}= \\
& \quad \frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right]\left[M_{1}\right]\left[M_{2}\right]  \tag{32}\\
& \\
& \quad+\frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right]\left[M_{2}\right]\left[M_{1}\right]-k_{4}\left[M_{3}^{+}\right]
\end{align*}
$$

Since pH is constant during the condensation step, we can write, in general,

$$
\begin{align*}
& \frac{d\left[M_{p}^{+}\right]}{d t}=\frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right] \sum_{i=1}^{p-1}\left[M_{i}\right]\left[M_{p-i}\right] \\
&  \tag{33}\\
& \quad-k_{4}\left[M_{p}^{+}\right] \\
& \begin{aligned}
\frac{d \sum_{j=2}^{\infty}\left[M_{j}^{+}\right]}{d t}= & \frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right] \sum_{j=2}^{\infty} \sum_{i=1}^{j-1}\left[M_{i}\right]\left[M_{j-i}\right] \\
& -k_{4} \sum_{j=2}^{\infty}\left[M_{j}^{+}\right]
\end{aligned}
\end{align*}
$$

Using the fact that all $M_{j}^{+}$are consumed at approximately the same rate at which they are formed (the pseudo-steady-state approximation), we have

$$
\begin{align*}
\frac{d \sum_{j=2}^{\infty}\left[M_{j}^{+}\right]}{d t} & =0 \Rightarrow \sum_{j=2}^{\infty}\left[M_{j}^{+}\right] \\
& =\frac{k_{3} K_{e}\left[\mathrm{H}^{+}\right]}{2 k_{4}} \sum_{j=2}^{\infty} \sum_{i=1}^{j-1}\left[M_{j}\right]\left[M_{j-i}\right]
\end{align*}
$$

The double summation in eq. (35) may be expressed as ${ }^{19}$

$$
\begin{equation*}
\sum_{j=2}^{\infty} \sum_{i=1}^{j-1}\left[M_{i}\right]\left[M_{j-i}\right]=\left[\sum_{i=1}^{\infty}\left[M_{i}\right]\right]^{2} \tag{36}
\end{equation*}
$$

Combining eqs. (25), (30), (35), and (36) yields
$\frac{d \sum_{j=1}^{\infty}\left[M_{j}\right]}{d t}$

$$
\begin{equation*}
=-\left[\frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right]+\frac{k_{t}}{2}\right]\left[\sum_{j=1}^{\infty}\left[M_{j}\right]\right]^{2} \tag{37}
\end{equation*}
$$

The number-average degree of condensation is defined as

$$
\begin{equation*}
\bar{D}_{n}=\frac{\sum i\left[M_{i}\right]}{\sum\left[M_{i}\right]}=\frac{M_{0}}{\sum\left[M_{i}\right]} \tag{38}
\end{equation*}
$$

in which $M_{0}$ is the total concentration of initial monomers. Therefore,

$$
\begin{equation*}
-\frac{M_{0} d \bar{D}_{n}}{\bar{D}_{n}^{2} d t}=-\left[\frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right]+\frac{k_{t}}{2}\right] \frac{M_{0}^{2}}{\bar{D} n^{2}} \tag{39}
\end{equation*}
$$

## EXPERIMENTAL

## Chemicals

Commercial formalin was supplied by SINA Chemical Co. and was used as the source of formaldehyde. Its concentration was $39.37 \%$ when analyzed using the sulfite method. ${ }^{21}$ The methanol content of formalin and its acidity were, respectively, $3.12 \%$ and 180 ppm using ASTM standards. ${ }^{22}$ Commercial urea with a nitrogen content of $45.8 \%$ was used. Urea was supplied by Shiraz Petrochemical Co. A modification of the sulfite method ${ }^{23}$ for formaldehyde solution was used for the determination of free formaldehyde.

## Resin Synthesis

Commercial formalin was poured into a beaker. Its pH was adjusted to 5.1 with 0.1 N NaOH . The required amount of urea was added to produce an initial $\mathrm{F} / \mathrm{U}$ ratio of 1.94 . The resulting solution was stirred for 4 min , and then the pH was adjusted to 8.5. This solution was charged to a threenecked reaction vessel placed in a water bath. The reaction vessel was equipped with a mechanical stirrer and a condenser. The reaction temperature was allowed to rise up to $89^{\circ} \mathrm{C}$ and free formaldehyde was periodically checked until equilibrium was reached (approximately about 35 min ). After this time, condensation reactions were initiated at constant temperature of $85 \pm 1^{\circ} \mathrm{C}$ by lowering the pH of the solution with formic acid to the desired value.

During condensation reactions, viscosity measurements were done using Ubbelohde viscometers at selected intervals. For this purpose, samples of about 25 mL were taken from the vessel and poured into a small beaker placed in an icewater bath to lower the temperature of the sample to $25^{\circ} \mathrm{C}$ as soon as possible. The procedure was the same for all experiments except for pH and time of condensation reactions.

## RESULTS AND DISCUSSION

Assuming that the relation between the logarithm of kinematic viscosity and the number-average


Figure 1 Kinematic viscosity changes during condensation reactions.
degree of condensation is linear (this is a proposal)

$$
\begin{equation*}
\ln v=A+B \bar{D} n \tag{39}
\end{equation*}
$$

we obtain

$$
\begin{equation*}
\frac{d(\ln v)}{B d t}=\left[\frac{k_{3} K_{e}}{2}\left[\mathrm{H}^{+}\right]+\frac{k_{t}}{2}\right] M_{0} \tag{40}
\end{equation*}
$$

Solving eq. (40) with the initial condition that at $t=0$ and $v=v_{0}$, we obtain the following result:

$$
\begin{align*}
\ln \left(\frac{v}{v_{0}}\right) & =\left[\frac{B M_{0} k_{3} K e\left[\mathrm{H}^{+}\right]}{2}+\frac{B M_{0} k_{t}}{2}\right] t  \tag{41}\\
v & =v_{0} \exp (b t) \tag{42}
\end{align*}
$$



Figure 2 Plot of $b$ versus hydronium ion concentration $\left[\mathrm{H}^{+}\right]$.

$$
\begin{equation*}
b=\frac{B M_{0} k_{3} K_{e}\left[\mathrm{H}^{+}\right]}{2}+\frac{B M_{0} k_{t}}{2} \tag{43}
\end{equation*}
$$

According to eq. (42), kinematic viscosity varies exponentially with time during condensation reactions. Also, $b$ is a linear function of hydronium ion concentration at constant temperature.

Equation (42) may be rewritten as

$$
\begin{equation*}
\ln v=\ln v_{0}+b t \tag{44}
\end{equation*}
$$

According to eq. (44), the logarithm of kinematic viscosity must be a linear function of time. In Figure 1 , the logarithm of kinematic viscosity is plotted against time for each experiment. The slope $b$ and intercept $\ln v_{0}$ for each experiment were calculated by the method of least squares and are shown in Table I. In Figure 2, $b$ was plotted

Table I Results of Regression Analysis

| Exp. <br> No. | pH During <br> Condensation Reactions | $v_{o}$ <br> $(\mathrm{cSt})$ | $b$ <br> $(1 / \mathrm{min})$ | Average Temperature <br> During Condensation <br> Reactions $\left.{ }^{\circ} \mathrm{C}\right)$ | Correlation <br> Coefficient |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.48 | 9.30 | .046 | 86.5 | 0.994 |
| 2 | 4.77 | 9.0 | .24 | 85.0 | 0.998 |
| 3 | 5.00 | 9.36 | .015 | 85.1 | 0.999 |
| 4 | 5.28 | 9.65 | .009 | 85.0 | 0.999 |
| 5 | 5.42 | 9.48 | .007 | 85.0 | 0.998 |
| 6 | 5.52 | 9.71 | .06 | 84.8 | 0.998 |
| 7 | 5.72 | 9.63 | .005 | 84.8 | 0.997 |
| 8 | 6.22 | 9.30 | .001 | 84.5 | 0.979 |

against hydronium ion concentration $\left[\mathrm{H}^{+}\right]$. The graph shows a linear relationship, as we expect from eq. (43).

Also as shown in Table I, $v_{0}$ is approximately constant. Its average value is 9.43 cSt . Finally, we present the following experimental correlation for the estimation of kinematic viscosity during condensation reactions for the foregoing experimental condition.
$\ln (v / 9.43)$

$$
\begin{equation*}
=\left(0.001586+1337.938 * 10^{-\mathrm{pH}}\right) * t \tag{45}
\end{equation*}
$$

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